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DEVELOPING CRITICAL AND LOGICAL THINKING SKILLS THROUGH THE USE OF NON-ROUTINE PROBLEMS AMONG PRE-SERVICE MATHEMATICS TEACHERS

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Abstract: The development of critical and logical thinking skills has become a central objective in mathematics teacher education, particularly in preparing pre-service teachers to engage with complex and non-standard problem situations. Mathematics learning that relies predominantly on routine procedures often limits opportunities for reasoning and reflective judgment. This study aims to examine how the use of non-routine problems contributes to the development of critical and logical thinking skills among pre-service mathematics teachers. The study employed a qualitative literature review methodology, synthesizing peer-reviewed research that addresses non-routine problem solving, reasoning processes, and teacher preparation in mathematics education. Data were analyzed thematically to identify recurring cognitive patterns, engagement characteristics, and reported outcomes related to critical and logical thinking development. The results indicate that non-routine problems promote prolonged engagement, evaluative reasoning, comparison of alternative strategies, and improved logical coherence in mathematical arguments. The discussion highlights that these outcomes reflect a shift from procedural orientation toward reasoning-centered mathematical thinking, supporting the role of non-routine problems as effective pedagogical tools in teacher education. In conclusion, the study affirms that systematic integration of non-routine problems in pre-service mathematics teacher education can strengthen critical and logical thinking skills. The findings suggest important implications for curriculum design and instructional practices aimed at preparing future mathematics teachers to foster higher-order thinking in classroom settings.

Keywords: non-routine problems, critical thinking, logical thinking, pre-service mathematics teachers, mathematics teacher education.

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INTRODUCTION

The increasing complexity of societal and technological challenges in the twenty-first century has intensified the demand for graduates who possess strong critical and logical thinking skills, particularly within the field of mathematics education. Mathematics is widely recognized as a discipline that nurtures reasoning, justification, and structured problem solving rather than mere computational proficiency [1]. Educational reforms across many countries emphasize higher-order thinking skills as essential learning outcomes, reflecting concerns about students' ability to apply mathematical knowledge in unfamiliar contexts [2]. Within this framework, pre-service mathematics teachers occupy a strategic position, as their cognitive competencies and beliefs shape future classroom practices [3]. However, numerous studies indicate that pre-service teachers often excel in routine procedural tasks while experiencing difficulty when confronted with non-standard or unfamiliar problems [4]. This tendency suggests an imbalance between procedural fluency and reasoning-oriented competence. As a consequence, teacher education institutions face mounting pressure to redesign instructional approaches that foster deeper forms of mathematical thinking. The integration of non-routine problems has gained attention as one potential response to this challenge.

Non-routine problems differ fundamentally from traditional textbook exercises because they do not provide explicit solution pathways and require learners to construct strategies independently [5]. Engagement with such problems encourages analysis, conjecture, and evaluation of multiple solution possibilities, which are central components of logical and critical thinking [6]. Research in mathematics education consistently shows that exposure to non-routine problems supports the development of reasoning habits and metacognitive awareness [7]. Despite these benefits, studies focusing on pre-service mathematics teachers reveal a persistent reliance on algorithmic reasoning, even after completing advanced coursework. This phenomenon suggests that traditional content-focused instruction may not adequately support the development of flexible thinking skills. In many teacher education programs, instructional time is dominated by coverage of mathematical topics rather than sustained problem exploration [8]. As a result, pre-service teachers often perceive non-routine problems as excessively difficult or disconnected from classroom realities. These findings underscore the need to investigate pedagogical strategies that systematically integrate non-routine problem solving into teacher preparation.

The importance of developing critical and logical thinking skills among pre-service mathematics teachers extends beyond academic theory and directly influences instructional practice. Teachers who lack confidence in solving non-routine problems may avoid presenting cognitively demanding tasks to their students, thereby limiting opportunities for meaningful mathematical engagement [9]. Comparative studies from education systems such as Singapore and Finland demonstrate that teacher preparation programs emphasizing problem-centered learning are associated with stronger instructional quality and student reasoning outcomes. These programs encourage teachers to facilitate inquiry, justify reasoning, and adapt instruction based on students' thinking. In contrast, contexts where teacher education relies heavily on procedural

training often struggle to implement reform-oriented mathematics curricula effectively. This contrast highlights how pre-service teachers' experiences with problem solving shape their future pedagogical choices. Developing critical and logical thinking during teacher preparation is therefore essential for sustaining instructional reform in mathematics education. Non-routine problems provide a meaningful context through which such cognitive development can occur.

From a scholarly perspective, critical and logical thinking in mathematics education has been examined through various theoretical lenses, including cognitive psychology and pedagogical content knowledge. Critical thinking involves evaluating assumptions, assessing arguments, and making reasoned judgments, while logical thinking emphasizes coherence, deduction, and structured reasoning [10]. Many existing studies measure these constructs using decontextualized assessments that do not fully capture how thinking unfolds during authentic learning activities. Research on non-routine problem solving offers a more situated approach, allowing scholars to observe reasoning processes as they emerge in real time [11]. However, empirical investigations that focus specifically on pre-service mathematics teachers remain relatively limited compared to studies on school students. This imbalance restricts understanding of how thinking skills develop within professional preparation contexts. By examining non-routine problem solving among pre-service teachers, this study contributes to a more comprehensive understanding of cognitive development in mathematics teacher education. It also responds to calls for research that bridges cognitive theory and instructional practice [12].

The primary objective of this study is to examine how the use of non-routine problems contributes to the development of critical and logical thinking skills among pre-service mathematics teachers. This objective includes analyzing changes in reasoning strategies, the coherence of logical arguments, and reflective processes during problem-solving activities. The study also seeks to explore pre-service teachers' perceptions of non-routine problems as part of their professional learning experience. By focusing on both cognitive outcomes and learning processes, the research provides a nuanced understanding of problem-based learning in teacher education. The central research question guiding this investigation is: How does the use of non-routine problems contribute to the development of critical and logical thinking skills among pre-service mathematics teachers? Addressing this question is expected to yield insights that inform curriculum design and instructional practice in teacher education programs. The findings aim to support the preparation of mathematics teachers who are capable of fostering higher-order thinking in their future classrooms.

LITERATURE REVIEW AND METHODOLOGY:

1. Critical and Logical Thinking in Mathematics Teacher Education

Critical and logical thinking have long been recognized as essential outcomes of mathematics education, particularly within teacher preparation programs. Critical thinking in mathematics involves the ability to evaluate information, question assumptions, and justify conclusions using evidence-based reasoning. Logical thinking, while closely related, emphasizes consistency, deductive reasoning, and the ability to construct coherent argumentation structures [13]. For pre-

service mathematics teachers, these skills are not only cognitive assets but professional necessities, as they underpin instructional decision-making and classroom discourse. Studies indicate that teachers' reasoning abilities significantly influence how mathematical ideas are represented and discussed in instructional settings [14]. When teachers demonstrate weak logical coherence, classroom interactions often prioritize answer-getting over conceptual understanding. This condition limits students' exposure to mathematical reasoning as a disciplined form of thinking. Consequently, the development of critical and logical thinking during teacher education has become a central concern in mathematics education research.

Despite widespread acknowledgment of their importance, empirical studies suggest that pre-service mathematics teachers frequently exhibit underdeveloped critical and logical thinking skills. Research shows that many teacher candidates rely heavily on memorized procedures and struggle to justify solution strategies or evaluate alternative approaches [15]. These tendencies are often attributed to prior learning experiences that emphasized procedural accuracy rather than reasoning processes. Teacher education programs sometimes replicate this pattern by prioritizing content coverage over reflective problem-solving activities. As a result, opportunities to cultivate higher-order thinking are limited. Scholars argue that without deliberate instructional interventions, critical and logical thinking do not develop automatically through exposure to mathematical content alone [16]. This body of literature highlights the need for pedagogical approaches that explicitly target reasoning development within pre-service teacher education.

2. Non-Routine Problems as a Medium for Developing Mathematical Reasoning

Non-routine problems occupy a central position in the literature on mathematical reasoning due to their capacity to disrupt algorithmic thinking. Such problems require learners to analyze unfamiliar situations, construct representations, and devise solution strategies without relying on predetermined procedures [17]. Fardah et al [18] argues that non-routine problem solving reveals learners' metacognitive control, strategic knowledge, and beliefs about mathematics. Engagement with these problems encourages exploration, hypothesis testing, and logical justification, all of which are core components of critical thinking. Empirical studies demonstrate that sustained exposure to non-routine problems improves learners' ability to reason flexibly and articulate mathematical arguments [19]. These findings support the view that non-routine problems function not merely as assessment tools but as learning environments that promote deep cognitive engagement. Their instructional value lies in making thinking processes visible and open to reflection.

In the context of teacher education, non-routine problems serve a dual function by shaping both cognitive competence and pedagogical perspective. Research indicates that pre-service teachers who engage regularly with non-routine problems develop greater appreciation for multiple solution strategies and student-centered instruction. However, studies also reveal resistance among teacher candidates who perceive such problems as impractical or excessively demanding [20]. This resistance often stems from limited prior exposure and a lack of confidence in navigating uncertainty. Without structured support, non-routine problems may be

viewed as obstacles rather than opportunities for learning. The literature suggests that carefully designed problem-based experiences can help pre-service teachers reconceptualize mathematics as a reasoning-oriented discipline. These insights underscore the importance of investigating how non-routine problem use influences thinking development within teacher preparation programs.

3. Empirical Studies on Problem-Based Learning in Pre-Service Mathematics Teachers

Empirical research on problem-based learning in mathematics education has consistently shown positive effects on reasoning and conceptual understanding. Studies involving school students report improvements in analytical thinking, strategy use, and mathematical communication when non-routine problems are integrated into instruction [21]. However, fewer studies focus specifically on pre-service mathematics teachers, despite their critical role in shaping future classroom practices. Existing research suggests that teacher candidates often approach problems with a solution-oriented mindset rather than a reasoning-oriented one [22]. This orientation limits opportunities for reflective thinking and critical evaluation. Intervention studies demonstrate that when pre-service teachers participate in structured problem-solving activities, they exhibit increased metacognitive awareness and improved logical coherence in explanations [23]. These findings indicate that problem-based learning holds promise for enhancing teacher cognition.

The literature also identifies methodological limitations in existing studies, including short intervention durations and reliance on self-reported data. Many investigations emphasize outcomes without sufficiently examining the reasoning processes that occur during problem solving. There is also limited exploration of how pre-service teachers' engagement with non-routine problems evolves over time. Scholars call for research designs that capture both cognitive development and learning experiences within authentic instructional contexts. Addressing these limitations is essential for advancing understanding of how problem-based approaches function in teacher education. By focusing on critical and logical thinking as observable processes, further research can contribute to more robust theoretical and practical insights. This study builds upon existing empirical work by examining non-routine problem use as a structured pathway for developing reasoning skills among pre-service mathematics teachers.

Methodology

Research Design

This study adopted a qualitative literature review design to explore how non-routine problems contribute to the development of critical and logical thinking skills among pre-service mathematics teachers. The qualitative approach was selected to allow an interpretive synthesis of existing research, focusing on conceptual patterns and pedagogical mechanisms rather than numerical aggregation. This design enabled the examination of how reasoning processes are described and analyzed across different educational contexts. By emphasizing meaning and context, the study sought to capture the complexity of cognitive development in mathematics teacher education. The design aligns with the objective of understanding learning processes rather than measuring causal effects. Qualitative literature review is particularly suitable for

synthesizing diverse theoretical and empirical perspectives. This approach provides a comprehensive conceptual foundation for the study.

Literature Search Strategy

The literature search was conducted systematically using established academic databases, including Scopus, Web of Science, and ERIC. Keywords and search strings were developed to reflect the core constructs of the study, such as "non-routine problems," "critical thinking," "logical thinking," and "pre-service mathematics teachers." Boolean operators were used to refine and combine search terms to ensure precision and breadth. The search focused on peer-reviewed journal articles, scholarly books, and conference proceedings to ensure academic quality. Publications written in English were prioritized to maintain conceptual consistency. The search process was iterative, allowing refinement of keywords based on emerging themes. This strategy ensured comprehensive identification of relevant literature.

Inclusion and Exclusion Criteria

Clear inclusion and exclusion criteria were established prior to literature selection. Studies were included if they explicitly addressed non-routine problem solving in mathematics education and discussed aspects of critical or logical thinking. Research focusing on pre-service mathematics teachers was prioritized, although foundational theoretical works were also included. Studies that addressed only routine problem practice or unrelated educational domains were excluded. Articles lacking clear methodological descriptions or theoretical grounding were not considered. This selection process ensured that the reviewed literature directly aligned with the research objectives. Applying these criteria enhanced the relevance and coherence of the reviewed sources.

Data Analysis Procedure

Data analysis was conducted using qualitative thematic analysis. Each selected study was read multiple times to identify recurring concepts, reasoning patterns, and pedagogical implications related to non-routine problem use. An inductive coding process was employed, allowing themes to emerge organically from the literature. Initial codes were grouped into broader thematic categories, such as cognitive processes, instructional strategies, and implementation challenges. Constant comparison across studies was used to refine and validate emerging themes. Analytical memos were maintained to document interpretive decisions and theoretical insights. This procedure enabled a structured and transparent synthesis of findings.

RESULTS AND THEIR ANALYSIS

1. Characteristics of Non-Routine Problem Engagement among Pre-Service Mathematics Teachers

The reviewed literature consistently reports that pre-service mathematics teachers demonstrate distinct engagement patterns when working with non-routine problems. Most studies describe initial responses characterized by uncertainty, prolonged problem interpretation, and repeated reformulation of given information. Rather than immediately applying formulas,

participants tend to pause to identify constraints, relationships, and possible representations of the problem situation. Several studies note that this phase involves trial-and-error reasoning, including the exploration of diagrams, symbolic expressions, and verbal explanations [24]. The duration of engagement with a single task is generally longer compared to routine exercises. This extended engagement reflects deeper cognitive involvement rather than inefficiency. The literature emphasizes that non-routine problems create a learning environment in which struggle becomes an inherent and visible component of mathematical activity.

Across multiple studies, pre-service teachers are reported to employ a variety of reasoning strategies during non-routine problem solving. These strategies include backward reasoning, pattern recognition, decomposition of complex problems into subproblems, and the testing of conjectures. The literature documents frequent shifts between strategies as participants reassess assumptions and revise approaches. Verbalization of thinking, either through written reflections or group discussion, appears prominently in qualitative descriptions [25]. Several studies highlight that reasoning processes are often non-linear, with participants revisiting earlier steps after encountering contradictions. This iterative reasoning process contrasts sharply with linear procedural execution commonly observed in routine problem contexts. The findings indicate that non-routine problems elicit diverse and dynamic forms of logical engagement.

The literature also reports observable differences in engagement levels based on prior exposure to problem-based learning. Pre-service teachers with limited experience in non-routine tasks exhibit higher levels of cognitive hesitation and reliance on peer support. In contrast, those with prior exposure demonstrate greater autonomy and persistence [26]. However, even experienced participants encounter moments of impasse, suggesting that non-routine problems consistently challenge existing cognitive structures. The reviewed studies describe engagement as both cognitively demanding and intellectually stimulating. These descriptions collectively portray non-routine problem engagement as an active, reflective, and sustained process. Such characteristics form the foundation for subsequent development of critical and logical thinking skills.

2. Development of Critical Thinking Processes through Non-Routine Problems

The literature documents multiple manifestations of critical thinking development during non-routine problem-solving activities. One recurring feature is the evaluation of assumptions, where pre-service teachers question the validity of initial interpretations and revise them based on emerging evidence. Studies report that participants frequently identify implicit constraints and reconsider problem conditions during solution attempts [27]. This evaluative behavior reflects critical examination rather than acceptance of given information at face value. Written reflections included in several studies show explicit acknowledgment of incorrect assumptions and justification for revised reasoning paths. Such behaviors are rarely reported in routine problem contexts. The findings suggest that non-routine problems create conditions that require sustained critical scrutiny.

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Another reported aspect of critical thinking is the comparison of alternative solution strategies. The literature describes instances where pre-service teachers generate multiple approaches and assess their feasibility. Participants are reported to weigh the efficiency, generalizability, and logical coherence of different solutions. In collaborative settings, peer discussion further stimulates critical evaluation as participants challenge each other's reasoning. Several studies document shifts in preference from computationally efficient strategies to conceptually transparent ones [28]. This indicates an emerging awareness of qualitative aspects of mathematical solutions. The ability to compare and justify alternatives is consistently identified as a critical thinking outcome.

The reviewed studies also highlight reflective judgment as a key component of critical thinking development. Pre-service teachers are reported to articulate reasons for accepting or rejecting certain strategies after completing problem-solving tasks. Reflection logs and interview excerpts reveal increased attention to reasoning quality rather than correctness alone. Participants frequently comment on how their thinking evolved throughout the task. Such reflective practices indicate movement toward self-regulated learning [5]. The literature portrays critical thinking as an observable process embedded within non-routine problem engagement rather than a separate skill measured externally.

3. Enhancement of Logical Reasoning Structures

Logical reasoning development is prominently reported across the reviewed studies. One recurring finding is improved coherence in mathematical arguments produced by pre-service teachers. Participants demonstrate increased ability to sequence reasoning steps logically and ensure consistency between assumptions and conclusions [6]. Written solutions increasingly include explicit statements linking each step to previous reasoning. The literature notes a reduction in unexplained procedural jumps. This structured presentation of reasoning reflects stronger logical organization.

Another reported outcome is the use of formal and informal deductive reasoning. Pre-service teachers are observed constructing arguments based on conditional reasoning, such as "if–then" statements, and validating conclusions through consistency checks. Several studies document the emergence of generalized reasoning, where participants move from specific cases to broader mathematical statements [7]. Logical counterexamples are also used to test conjectures. These behaviors indicate an expansion of logical reasoning beyond computation. The findings show that non-routine problems provide a context for practicing logical justification in meaningful ways.

The literature further reports increased awareness of logical validity. Participants demonstrate sensitivity to contradictions and logical gaps within their own reasoning. In several studies, pre-service teachers explicitly identify errors not due to calculation mistakes but due to flawed logical assumptions. This awareness leads to revisions of argument structure rather than surface-level corrections [29]. The presence of logical self-monitoring is consistently

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documented as a result of sustained non-routine problem engagement. These findings illustrate the role of non-routine problems in strengthening internal logical control mechanisms.

4. Synthesis of Reported Outcomes across Studies

Across the reviewed literature, consistent outcome patterns emerge regarding the impact of non-routine problems on pre-service mathematics teachers' thinking skills. Studies report convergence in observed behaviors related to engagement, critical evaluation, and logical structuring. Although contexts and methodologies vary, similar cognitive processes are described across institutional and cultural settings [30]. This consistency suggests that non-routine problems function as a stable instructional stimulus for higher-order thinking development. The synthesis reveals common thematic outcomes rather than isolated effects. These outcomes are observable in both individual and collaborative learning environments.

The reviewed studies also report challenges associated with non-routine problem implementation. Initial resistance, increased cognitive load, and time constraints are frequently mentioned. However, these challenges are described alongside gradual adaptation and increased tolerance for uncertainty. Over time, participants demonstrate improved persistence and strategic flexibility [31]. The literature documents a shift from answer-oriented behavior to reasoning-oriented engagement. These reported changes are consistent across longitudinal and short-term intervention studies.

To consolidate the findings, thematic outcomes identified in the literature are summarized in Table 1. This table presents recurring cognitive indicators associated with non-routine problem engagement. The synthesis highlights observable patterns rather than evaluative judgments. The table serves as an empirical representation of the qualitative results reported across studies.

Table 1. Synthesized Outcomes of Non-Routine Problem Engagement in Pre-Service Mathematics Teachers

Theme Area	Observed Indicators	Description in Literature
Engagement Characteristics	Prolonged analysis, strategy shifts	Extended time on task, iterative reasoning
Critical Thinking	Assumption evaluation, comparison of strategies	Questioning initial interpretations and justifying alternatives
Logical Reasoning	Coherent argumentation, deductive structure	Explicit reasoning steps and logical consistency
Reflective Practice	Self-evaluation, reasoning revision	Awareness of reasoning quality and logical validity

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Discussion

The findings of this qualitative literature review demonstrate that non-routine problems play a substantial role in shaping how pre-service mathematics teachers engage with mathematical reasoning. The observed patterns of prolonged engagement, strategy exploration, and iterative problem reformulation indicate a shift away from procedural execution toward meaning-oriented problem solving. This aligns with prior literature describing non-routine problems as environments that surface learners' thinking processes rather than conceal them behind algorithmic fluency. The results suggest that cognitive struggle, often perceived negatively in traditional instruction, functions as a productive condition for reasoning development in teacher education contexts. Pre-service teachers' willingness to persist through uncertainty reflects growing intellectual autonomy [11]. Such engagement patterns indicate that non-routine problems create conditions under which reasoning becomes an explicit object of learning. This finding reinforces the pedagogical value of designing tasks that resist immediate procedural resolution.

The development of critical thinking processes observed in the reviewed studies highlights how non-routine problems encourage evaluative reasoning. Pre-service teachers' tendencies to question assumptions, reconsider interpretations, and justify strategic choices illustrate critical engagement with mathematical content. These behaviors resonate with conceptualizations of critical thinking as reflective judgment and reasoned decision-making within disciplinary contexts [12]. The literature indicates that such thinking does not emerge from exposure to content alone but requires learning situations that demand justification and reflection. Non-routine problems appear to fulfill this function by disrupting habitual solution patterns. The comparison of alternative strategies further demonstrates an emerging awareness of qualitative dimensions of mathematical reasoning. This suggests that non-routine problems contribute to developing discernment rather than merely expanding procedural repertoire.

The enhancement of logical reasoning structures reported across studies underscores the role of non-routine problems in fostering coherent mathematical argumentation. The increased use of explicit reasoning sequences, conditional logic, and consistency checks reflects strengthened internal logical control. These outcomes correspond with descriptions in the literature that position logical thinking as the capacity to maintain coherence between assumptions, processes, and conclusions. Non-routine problems require pre-service teachers to construct reasoning chains rather than execute isolated steps [17]. The emergence of generalized reasoning and counterexample testing further indicates maturation of logical structures. Such developments are particularly significant for future teachers, whose instructional clarity depends on logical coherence. The findings suggest that sustained engagement with non-routine problems supports the internalization of logical discipline within mathematical thinking.

The synthesis of outcomes across diverse studies reveals a high degree of consistency in reported cognitive developments, despite variation in educational contexts and research designs. This consistency strengthens the credibility of non-routine problems as a pedagogical medium

for cultivating higher-order thinking in pre-service mathematics teachers [21]. While challenges such as initial resistance and increased cognitive load are frequently reported, the literature describes these as transitional rather than prohibitive. Over time, pre-service teachers demonstrate greater tolerance for ambiguity and increased strategic flexibility. This progression suggests that discomfort is an integral part of reasoning development rather than an indicator of instructional failure [32]. The shift from answer-oriented to reasoning-oriented engagement represents a fundamental transformation in how mathematics is approached. Such transformation aligns with broader goals of reform-oriented mathematics education.

Taken together, the discussion indicates that non-routine problems contribute meaningfully to the development of critical and logical thinking by reshaping engagement patterns, reasoning practices, and reflective awareness among pre-service mathematics teachers [33]. The findings suggest that teacher education programs seeking to cultivate higher-order thinking should consider non-routine problems as core instructional components rather than supplementary activities. Embedding these problems systematically within coursework may help future teachers internalize reasoning-centered views of mathematics. This has implications for curriculum design, instructional practice, and professional identity formation in mathematics teacher education [34]. The discussion reinforces the view that thinking skills develop through sustained interaction with cognitively demanding tasks. As such, non-routine problems represent a viable pedagogical pathway for preparing mathematics teachers capable of fostering critical and logical thinking in their future classrooms.

CONCLUSIONS

This study concludes that the use of non-routine problems constitutes a meaningful approach for fostering critical and logical thinking skills among pre-service mathematics teachers. The qualitative synthesis of the literature demonstrates that engagement with non-routine problems reshapes how future teachers interact with mathematical tasks, shifting emphasis from procedural execution toward reasoning, justification, and reflection. Pre-service teachers consistently exhibit increased persistence, strategic flexibility, and coherence in logical argumentation when exposed to non-standard problem contexts. These outcomes indicate that non-routine problems function as cognitive catalysts that surface and develop higher-order thinking processes. The findings affirm that reasoning skills are not incidental by-products of content learning but emerge through sustained engagement with cognitively demanding tasks. This conclusion underscores the pedagogical value of positioning non-routine problems at the core of mathematics teacher education.

The implications of these findings are significant for mathematics teacher education programs and curriculum design. Integrating non-routine problems systematically within pre-service coursework can support the development of reasoning-oriented instructional identities. Teacher educators may use non-routine problems not only as learning tasks but also as tools for modeling pedagogical practices that emphasize inquiry and justification. Such integration has the potential to enhance future teachers' confidence in facilitating complex mathematical discussions in classrooms. At the institutional level, the findings suggest the need to reconsider

assessment practices that prioritize procedural accuracy over reasoning quality. Embedding reflective components alongside non-routine problem solving may further strengthen critical and logical thinking development. These implications highlight the role of curriculum structure in shaping cognitive and professional outcomes.

Based on the conclusions and implications, several recommendations can be proposed for future practice and research. Teacher education programs are encouraged to design sustained and scaffolded non-routine problem experiences rather than isolated activities. Professional development initiatives may also incorporate non-routine problem solving to support in-service teachers' reasoning development. Future research should examine longitudinal effects of non-routine problem integration on instructional practices during teaching internships and early career stages. Investigating contextual factors, such as institutional support and instructional culture, may further illuminate conditions that enhance or constrain reasoning development. These recommendations aim to extend understanding of how non-routine problems contribute to the preparation of mathematics teachers capable of fostering critical and logical thinking in diverse educational settings.

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